**(2 slide)**

Let us consider a chain of connected and singularly perturbed oscillators with a delay:

(1)

Also for it there are determined the following initial conditions:

(initial conditions for (1))

Functions are smooth.

**(3 slide)**

In these articles there were applied the following substitutions

**(4 slide)**

(substitutions)

Here is a solution of this differential equation:

(equation)

are new variables. By means of these substitutions system (1) is transformed to the impulsive system of differential equations:

**(5 slide)**

(2)

**(6 slide)**

Let us consider a map with initial conditions:

(3)

is the first approximation of stable cycle for single oscillator of system (1). For this map there was proved this theorem:

(theorem 1)

In other words for research of relaxational cycles of systems (1) and (2) we can just research stable points of map (3).

**(7 slide)**

So the research of map (3) was carried out by means of special software written in C++.

All calculations are performed on a large number of independent streams of CPU. So the program uses the technology of parallel calculations OpenMP. Given numerical results are shown as a phase portrait of map (3). To visualize data there was developed a special application using the framework Qt. Also there was wrote a script in Python to solve the problem of parsing data.

**(8 slide)**

An asymptotic analysis of this map shows that it has at least stable points with certain coordinates. Moreover zero point is stable for any values of . But we don’t except the existence of auxiliary stable points with coordinates, which aren’t determined by formulas in the articles.

(map)

So my research task was the search of values of initial parameters, for which auxiliary stable points exist. Also in the case of 3 oscillators, there are researched bifurcations in a phase space of map (3).

(grant)

This research was supported by the Russian Science Foundation.

**(9 slide)**

In the case of bidirectional wave with Neuman boundary conditions chain of oscillators looks like this:

(system)

**(10 slide)**

and impulsive system is of the form presented on this slide:

(impulsive system)

**(11 slide)**

On coordinate plane of parameters and there are regions and curves

(picture)

The most important element for building of these regions is line described by formula . Curves and are symmetric relative to line and touch each other in point . These curves are borders of region . Also in point curve traces to line . It permits to determine region . Region is doubly connected. Line described by formula . It’s one of conditions on and in system (1).

The borders of regions depend on maximal amount of stable points, which are detected there for map (3).

**(12 slide)**

For values of parameters and from region it is possible to exist 5 stable points

(picture)

**(13 slide)**

In regions and it is possible to exist 7

(picture)

**(14 slide)**

and 6 stable points, respectively

(picture)

On these phase portraits blue points are unstable, are stable, black lines are separatrixes, grey lines are some phase curves.

**(15 slide)**

In these articles there are examples of different bifurcations for certain values of initial parameters.

**(16 slide)**

In the case of bidirectional wave with periodic boundary conditions chain of oscillators looks like this:

(system)

**(17 slide)**

and impulsive system is of the form presented on this slide:

(impulsive system)

**(18 slide)**

Here you can see the regions with the same bifurcations for this case

(picture)

**(19 slide)**

For values of parameters and from regions and it is possible to exist 7 stable points for map (3). The difference between these cases is the types of unstable points .

(pictures)

**(20 slide)**

So in region there is an unstable manifold around zero balance state instead of unstable points .

(picture)

Every point of this manifold is an unstable state.

(pictures)

**(21 slide)**

In this article there are examples of different bifurcations for certain values of initial parameters.

**(22 slide)**

In the case of unidirectional wave chain of oscillators looks like this:

(system)

**(23 slide)**

and impulsive system is of the form presented on this slide:

(impulsive system)

**(24 slide)**

Here you can see the regions with the same bifurcations for this case

(picture)

Line comes nearer to curve , when parameter increases.

**(25 slide)**

For every region and , as in this articles

(articles)

there were given all possible bifurcations in a phase space of map (3). The difference is the bifurcational value of parameter , when the bifurcation occures. So let us consider more detailed results.

**(26 slide)**

Here you can see the sequence of bifurcations for such fixed parameters and from

(parameters)

when increases.

(pictures)

**(27 slide)**

Here you can see the sequence of bifurcations for and from , when increases.

(parameters & pictures)

The movement on manifold is clockwise.

**(28 slide)**

Here you can see the sequence of bifurcations for and from , when increases.

**(29 slide)**

Al last, if we come back to initial task with system (1), we can find out, that every stable point of map (3) is satisfied to relaxational cycle of system (1) and every stable manifold with movement is satisfied to stable torus.

**(30 slide)**

The first fact was proved in these article and the second fact is intuitively obvious, but the proof of this statement demands of mathematical manipulations